Unusual formations of the free electromagnetic field in vacuum

2002 J. Phys. A: Math. Gen. 35 8043

(http://iopscience.iop.org/0305-4470/35/38/307)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **35** (2002) 8043–8053 PII: S0305-4470(02)37984-8

Unusual formations of the free electromagnetic field in vacuum

Andrew E Chubykalo and Augusto Espinoza

Escuela de Física, Universidad Autónoma de Zacatecas, Apartado Postal C-580 Zacatecas 98068, ZAC, Mexico

E-mail: agarrido@logicnet.com.mx and andrew@logicnet.com.mx

Received 11 June 2002
Published 12 September 2002
Online at stacks.iop.org/JPhysA/35/8043

Abstract
It is shown that there are exact solutions of the free Maxwell equations (FME) in vacuum allowing the existence of stable spherical formations of the free magnetic field and ring-like formations of the free electric field. It is detected that the form of these spheres and rings does not change with time in vacuum. It is shown that these *convergent* solutions are the result of the interference of some *divergent* solutions of the FME. One can surmise that these electromagnetic formations correspond to Kapitsa's hypothesis about interference origin and the structure of a *fireball*.

PACS numbers: 03.50.−z, 03.50.De

1. Introduction

It is the generally accepted opinion that solutions of the free Maxwell equations (FME) are well studied and do not boil down to any surprises. Nevertheless, we will show in the next sections that such mathematically well-known solutions (see, e.g., [1], where general solutions of the Maxwell equations were obtained) lead, however, to the existence of rather unusual and unexpected electromagnetic formations in vacuum such as closed spherical magnetic surfaces (*without* an electric field on these surfaces and where the magnetic field is tangential and its intensity depends on time) and ring-like formations of the electric field (*without* a magnetic field at all points of the ring and where the electric field is tangential and depends on time). We will also show that these formations do not change their form with time in vacuum.

2. An unusual solution of the free Maxwell equations

We found that a certain class of exact solutions of the FME

$$\operatorname{div}\mathbf{E} = 0 \tag{1}$$

$$\text{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} \tag{2}$$

$$\text{div}\,\mathbf{B} = 0 \tag{3}$$

$$\text{rot}\,\mathbf{B} = \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} \tag{4}$$

exists which has some unexpected characteristics. The present work is devoted to the research of such solutions.

We shall look for solutions of the system FME as follows:

$$\mathbf{E}(\boldsymbol{r}, t) = \boldsymbol{e}(\boldsymbol{r})\psi(t) \qquad \text{and} \qquad \mathbf{B}(\boldsymbol{r}, t) = \boldsymbol{b}(\boldsymbol{r})\chi(t) \tag{5}$$

where $\psi(t)$ and $\chi(t)$ are some functions of time, vector $\boldsymbol{e}$ is a polar vector and $\boldsymbol{b}$ is an axial one.

And so substituting (5) into the FME we obtain

$$\text{div}\,\boldsymbol{e} = 0 \tag{6}$$

$$\text{rot}\,\boldsymbol{e} = -\frac{1}{c}\frac{\chi'}{\psi}\boldsymbol{b} \tag{7}$$

$$\text{div}\,\boldsymbol{b} = 0 \tag{8}$$

$$\text{rot}\,\boldsymbol{b} = \frac{1}{c}\frac{\psi'}{\chi}\boldsymbol{e}. \tag{9}$$

It is obvious that these equations are consistent if and only if

$$-\frac{\chi'}{\psi} = \Omega_1 \qquad \text{and} \qquad \frac{\psi'}{\chi} = \Omega_2 \tag{10}$$

where a prime indicates a derivative with respect to time, Ω_1 and Ω_2 are arbitrary constants.

In order to obtain solutions of this system with three constants only, and to obtain sinusoidal solutions, we propose that $\Omega_1 = \Omega_2 = \Omega$. Thus, the general solution of the system (10) is

$$\chi(t) = \mathcal{A}\cos(\Omega t - \eta) \qquad \text{and} \qquad \psi(t) = \mathcal{A}\sin(\Omega t - \eta) \tag{11}$$

where $\mathcal{A}$ and η are arbitrary constants, and equations for $\boldsymbol{e}$ and $\boldsymbol{b}$ become

$$\nabla \times \boldsymbol{e} = \frac{\Omega}{c}\boldsymbol{b} \qquad \text{and} \qquad \nabla \times \boldsymbol{b} = \frac{\Omega}{c}\boldsymbol{e}. \tag{12}$$

In order to solve this system, let us first note that formally summing two equations (12) we obtain

$$\nabla \times (\boldsymbol{e} + \boldsymbol{b}) = \frac{\Omega}{c}(\boldsymbol{e} + \boldsymbol{b}) \qquad \text{or} \qquad \nabla \times \boldsymbol{a} = \frac{\Omega}{c}\boldsymbol{a}. \tag{13}$$

So, first we resolve equation (13) with respect to $\boldsymbol{a}$, and then we obtain from the vector $\boldsymbol{a}$ (which, obviously, has no polarity) the polar vector $\boldsymbol{e}$ and the axial vector $\boldsymbol{b}$. Actually, one can express the polar and axial parts of any vector without polarity, in general, as follows:

$$\boldsymbol{e}(\boldsymbol{r}) = \tfrac{1}{2}[\boldsymbol{a}(\boldsymbol{r}) - \boldsymbol{a}(-\boldsymbol{r})] \tag{14}$$

and

$$\boldsymbol{b}(\boldsymbol{r}) = \tfrac{1}{2}[\boldsymbol{a}(\boldsymbol{r}) + \boldsymbol{a}(-\boldsymbol{r})]. \tag{15}$$

Now, if we calculate a rotor of both parts of equations (14), (15) we can be satisfied that the system (12) is fulfilled:

$$\nabla \times \boldsymbol{e}(\boldsymbol{r}) = \frac{1}{2}[\nabla \times \boldsymbol{a}(\boldsymbol{r}) - \nabla \times \boldsymbol{a}(-\boldsymbol{r})] = \frac{1}{2}\left[\frac{\Omega}{c}\boldsymbol{a}(\boldsymbol{r}) + \frac{\Omega}{c}\boldsymbol{a}(-\boldsymbol{r})\right] = \frac{\Omega}{c}\boldsymbol{b}(\boldsymbol{r}) \tag{16}$$

and

$$\nabla \times \boldsymbol{b}(\boldsymbol{r}) = \frac{1}{2}[\nabla \times \boldsymbol{a}(\boldsymbol{r}) + \nabla \times \boldsymbol{a}(-\boldsymbol{r})] = \frac{1}{2}\left[\frac{\Omega}{c}\boldsymbol{a}(\boldsymbol{r}) - \frac{\Omega}{c}\boldsymbol{a}(-\boldsymbol{r})\right] = \frac{\Omega}{c}\boldsymbol{e}(\boldsymbol{r}). \tag{17}$$

Here, we take into account that after inverting the coordinates, the equation $\nabla \times \boldsymbol{a}(\boldsymbol{r}) = \frac{\Omega}{c}\boldsymbol{a}(\boldsymbol{r})$ becomes $-\nabla \times \boldsymbol{a}(-\boldsymbol{r}) = \frac{\Omega}{c}\boldsymbol{a}(-\boldsymbol{r})$. Thus, one can see that if we find $\boldsymbol{a}$ as a solution of equation (13) it means that we find $\boldsymbol{e}$ and $\boldsymbol{b}$ as solutions of the system (12).

Equation (13) has already been solved in the literature (see, e.g., [2, 3]).

And so, the solution of equation (13) in the spherical system of coordinates is[1]

$$\boldsymbol{a} = \mathcal{D}\left\{\frac{2\alpha}{r^3}\cos\theta\right\}\boldsymbol{e}_r + \mathcal{D}\left\{\frac{\gamma}{r^3}\sin\theta\right\}\boldsymbol{e}_\theta + \mathcal{D}\left\{\frac{\Omega\alpha}{cr^2}\sin\theta\right\}\boldsymbol{e}_\varphi. \tag{18}$$

Finally, separating vectors $\boldsymbol{e}$ and $\boldsymbol{b}$ we obtain the solution of the system (12) expressed by components (Cartesian and spherical ones):

$$\boldsymbol{e} = \mathcal{D}\left\{-\frac{\alpha\Omega y}{cr^3},\quad \frac{\alpha\Omega x}{cr^3},\quad 0\right\} = \frac{\Omega\alpha\sin\theta}{cr^2}\mathcal{D}\boldsymbol{e}_\varphi \tag{19}$$

and

$$\boldsymbol{b} = \mathcal{D}\left\{\frac{\beta xz}{r^5},\quad \frac{\beta yz}{r^5},\quad \frac{2\alpha}{r^3} - \frac{\beta(x^2+y^2)}{r^5}\right\} = \frac{2\alpha\cos\theta}{r^3}\mathcal{D}\boldsymbol{e}_r + \frac{\gamma\sin\theta}{r^3}\mathcal{D}\boldsymbol{e}_\theta \tag{20}$$

where

$$\alpha = \cos\left(\frac{\Omega r}{c} - \delta\right) + \frac{\Omega r}{c}\sin\left(\frac{\Omega r}{c} - \delta\right)$$

$$\beta = 3\alpha - \frac{\Omega^2 r^2}{c^2}\cos\left(\frac{\Omega r}{c} - \delta\right) \qquad \text{and} \qquad \gamma = \beta - 2\alpha.$$

Let us now write the solution (5) in explicit form, taking into account equations (11), (19) and (20):

$$\mathbf{E} = \left[\frac{\Omega\alpha\sin\theta}{cr^2}\mathcal{D}\boldsymbol{e}_\varphi\right]\sin(\Omega t - \eta) \tag{21}$$

and

$$\mathbf{B} = \left[\frac{2\alpha\cos\theta}{r^3}\mathcal{D}\boldsymbol{e}_r + \frac{\gamma\sin\theta}{r^3}\mathcal{D}\boldsymbol{e}_\theta\right]\cos(\Omega t - \eta) \tag{22}$$

where δ and η are arbitrary constants.

It follows from solutions (21), (22) that the necessary (not sufficient!) condition in order for these solutions to not diverge in $r = 0$ is

$$\alpha(0) = \left\{\cos\left(\frac{\Omega r}{c} - \delta\right) + \frac{\Omega r}{c}\sin\left(\frac{\Omega r}{c} - \delta\right)\right\}\Bigg|_{r=0} = 0 \quad\Longrightarrow\quad \cos\delta = 0 \quad\Longrightarrow$$

$$\delta = \left(n + \frac{1}{2}\right)\pi \qquad \text{where} \quad n = 0, \pm 1, \pm 2, \ldots.$$

[1] $\mathcal{D}$ is a dimensional constant $[\mathcal{D}] = \mathtt{M}^{1/2}\mathtt{L}^{5/2}\mathtt{T}^{-1}$.

In order to satisfy oneself that the solutions (21), (22) converge, one can calculate the following limits[2] for $\delta = \frac{\pi}{2}$:

$$\lim_{r\to 0}\frac{\alpha}{r^2} = 0 \qquad \lim_{r\to 0}\frac{\alpha}{r^3} = \frac{\Omega^3}{3c^3} \qquad \lim_{r\to 0}\frac{\gamma}{r^3} = -\frac{2\Omega^3}{3c^3} \tag{23}$$

and the corresponding limits for **E**, **B** and the energy density $w = \frac{E^2+B^2}{8\pi}$ are

$$\lim_{r\to 0}\mathbf{E} = 0 \qquad \lim_{r\to 0}\mathbf{B} = \frac{2\mathcal{D}\Omega^3\cos(\Omega t)}{3c^3}\mathbf{k} \qquad \lim_{r\to 0} w = \frac{\mathcal{D}^2\Omega^6}{18\pi c^6}\cos^2(\Omega t) \tag{24}$$

where **k** is the z-coordinate of the Cartesian system.

The constant η just defines an initial wave phase of the fields **E** and **B**. So, without loss of generality, we can just write one non-divergent solution for $\delta = \frac{\pi}{2}$, $\eta = 0$ as

$$\mathbf{E} = \mathcal{D}\left[\frac{\alpha\Omega\sin\theta}{cr^2}\boldsymbol{e}_\varphi\right]\sin(\Omega t) \qquad \mathbf{B} = \mathcal{D}\left[\frac{2\alpha\cos\theta}{r^3}\boldsymbol{e}_r + \frac{\gamma\sin\theta}{r^3}\boldsymbol{e}_\theta\right]\cos(\Omega t) \tag{25}$$

where

$$\alpha = -\frac{\Omega r}{c}\cos\left(\frac{\Omega r}{c}\right) + \sin\left(\frac{\Omega r}{c}\right) \qquad \text{and} \qquad \gamma = \alpha - \frac{\Omega^2 r^2}{c^2}\sin\left(\frac{\Omega r}{c}\right). \tag{26}$$

Note that the solution (25) can be found directly from the general solution of the Maxwell equations obtained by Mie [1].

Generally speaking, taking into account (19) and (20) one can see that an infinity of divergent solutions for **E** and **B** exist as well as convergent ones. Curious enough, but the convergency of these solutions is defined by the constant δ. Solutions converge in $r = 0$ if and only if

$$\delta = \left(n + \tfrac{1}{2}\right)\pi \qquad \text{where} \quad n = 0, \pm 1, \pm 2, \ldots. \tag{27}$$

We show that the convergent solution (25)[3] is represented as an interference of divergent ones.

After simple algebraic transformation one can represent the solution (25) as a superposition of two waves spreading in opposite directions at every point:

$$\mathbf{E}_c = \mathbf{E}_{(\rightarrow)} + \mathbf{E}_{(\leftarrow)} \qquad \text{and} \qquad \mathbf{B}_c = \mathbf{B}_{(\rightarrow)} + \mathbf{B}_{(\leftarrow)} \tag{28}$$

where

$$\mathbf{E}_{(\rightarrow)} = \frac{\Omega\sin\theta}{2cr^2}\left[\cos\left(\frac{\Omega r}{c} - \Omega t\right) + \frac{\Omega r}{c}\sin\left(\frac{\Omega r}{c} - \Omega t\right)\right]\mathcal{D}\boldsymbol{e}_\varphi \tag{29}$$

$$\mathbf{E}_{(\leftarrow)} = -\frac{\Omega\sin\theta}{2cr^2}\left[\cos\left(\frac{\Omega r}{c} + \Omega t\right) + \frac{\Omega r}{c}\sin\left(\frac{\Omega r}{c} + \Omega t\right)\right]\mathcal{D}\boldsymbol{e}_\varphi \tag{30}$$

$$\begin{aligned}\mathbf{B}_{(\rightarrow)} = {} & \frac{\cos\theta}{r^3}\left[\sin\left(\frac{\Omega r}{c} - \Omega t\right) - \frac{\Omega r}{c}\cos\left(\frac{\Omega r}{c} - \Omega t\right)\right]\mathcal{D}\boldsymbol{e}_r \\ & + \frac{\sin\theta}{2r^3}\left[-\frac{\Omega r}{c}\cos\left(\frac{\Omega r}{c} - \Omega t\right) + \left(1 - \frac{\Omega^2 r^2}{c^2}\right)\sin\left(\frac{\Omega r}{c} - \Omega t\right)\right]\mathcal{D}\boldsymbol{e}_\theta\end{aligned} \tag{31}$$

[2] We calculate these limits expanding α and γ in series of powers of r.

[3] We designate it by $\mathbf{E}_c$ and $\mathbf{B}_c$ in this section.

$$\mathbf{B}_{(\to)} = \frac{\cos\theta}{r^3}\left[\sin\left(\frac{\Omega r}{c} + \Omega t\right) - \frac{\Omega r}{c}\cos\left(\frac{\Omega r}{c} + \Omega t\right)\right]\mathcal{D}\boldsymbol{e}_r$$
$$+ \frac{\sin\theta}{2r^3}\left[-\frac{\Omega r}{c}\cos\left(\frac{\Omega r}{c} + \Omega t\right) + \left(1 - \frac{\Omega^2 r^2}{c^2}\right)\sin\left(\frac{\Omega r}{c} + \Omega t\right)\right]\mathcal{D}\boldsymbol{e}_\theta. \quad (32)$$

Labourless calculation also shows that

$$\mathbf{E}_{(\to)} = \tfrac{1}{2}(-\mathbf{E}_d + \mathbf{E}_c) \qquad \mathbf{E}_{(\leftarrow)} = \tfrac{1}{2}(\mathbf{E}_d + \mathbf{E}_c) \quad (33)$$

and

$$\mathbf{B}_{(\to)} = \tfrac{1}{2}(-\mathbf{B}_d + \mathbf{B}_c) \qquad \mathbf{B}_{(\leftarrow)} = \tfrac{1}{2}(\mathbf{B}_d + \mathbf{B}_c) \quad (34)$$

where $\mathbf{E}_d$, $\mathbf{B}_d$ are divergent solutions of the system (1)–(4):

$$\mathbf{E}_d = \mathcal{D}\left[\frac{\alpha_d \Omega \sin\theta}{cr^2}\boldsymbol{e}_\varphi\right]\cos(\Omega t) \qquad \mathbf{B}_d = \mathcal{D}\left[\frac{2\alpha_d\cos\theta}{r^3}\boldsymbol{e}_r + \frac{\gamma_d\sin\theta}{r^3}\boldsymbol{e}_\theta\right]\sin(\Omega t) \quad (35)$$

and

$$\alpha_d = \cos\left(\frac{\Omega r}{c}\right) + \frac{\Omega r}{c}\sin\left(\frac{\Omega r}{c}\right) \qquad \text{and} \qquad \gamma_d = \alpha - \frac{\Omega^2 r^2}{c^2}\sin\left(\frac{\Omega r}{c}\right).$$

It is obvious that the functions $\mathbf{E}_{(\to)}$, $\mathbf{E}_{(\leftarrow)}$, $\mathbf{B}_{(\to)}$, $\mathbf{B}_{(\leftarrow)}$ diverge in $r = 0$ and they are also solutions of the FME.

3. Stable electromagnetic spheres and rings in vacuum as a consequence of the solution (25)

As we will show below, the solution (25) of the FME leads to the existence of unusual spherical formations of the free electromagnetic field.

3.1. Some details of the energy distribution in the field (25)

Let us write, after some transformations, the expression for the energy density for the solution (25). One can show that the energy density contains both time-independent and time-dependent parts:

$$w = \frac{E^2 + B^2}{8\pi} = \frac{\mathcal{D}^2}{16\pi}\left\{\frac{\Omega^2\alpha^2}{c^2 r^4}\sin^2\theta + \left[\frac{4\alpha^2}{r^6}\cos^2\theta + \frac{\gamma^2}{r^6}\sin^2\theta\right]\right\}$$
$$+ \frac{\mathcal{D}^2}{16\pi}\left\{\left[\frac{4\alpha^2}{r^6}\cos^2\theta + \frac{\gamma^2}{r^6}\sin^2\theta\right] - \frac{\Omega^2\alpha^2}{c^2 r^4}\sin^2\theta\right\}\cos(2\Omega t). \quad (36)$$

Let us find from (36) the *locus* where w does not depend on t. It is obvious that the *loci* are

(1) along the axis Z at the points where $\tan\left(\frac{\Omega z}{c}\right) = \frac{\Omega z}{c}(\theta = 0, \pi; \alpha = 0)$;
(2) at surfaces where r satisfies the equation $\gamma^2 = \alpha^2\left(\frac{\Omega^2 r^2}{c^2} - 4\cot^2\theta\right)$. One can see the cross-section of these surfaces in figure 3 (discontinuous curves)[4]

Now we calculate the electromagnetic energy $\mathcal{E}$ within a sphere of radius R with the centre at the coordinate origin:

$$\mathcal{E}_\oplus = \int_0^R \mathrm{d}r \int_0^\pi \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\varphi\, r^2 \sin\theta\, w(r,\theta,\varphi,t) = \mathcal{E}(R) + \mathcal{E}(R,t) \quad (37)$$

[4] All figures in this work were performed in the program 'Mathematica-4.0'.

where

$$\mathcal{E}(R) = \frac{\mathcal{D}^2}{6R^3}\left[\frac{\Omega^4 R^4}{c^4} - \frac{\Omega^2 R^2}{c^2}\sin^2\left(\frac{\Omega R}{c}\right) - \alpha^2\right] \tag{38}$$

$$\mathcal{E}(R,t) = -\frac{\mathcal{D}^2}{6R^3}\alpha\gamma\cos(2\Omega t). \tag{39}$$

Here $\alpha = -\frac{\Omega R}{c}\cos\left(\frac{\Omega R}{c}\right) + \sin\left(\frac{\Omega R}{c}\right)$ and $\gamma = \alpha - \frac{\Omega^2 R^2}{c^2}\cos\left(\frac{\Omega R}{c}\right)$.

One can show from equation (39) that electromagnetic energy within spheres of radii R which are solutions of the equations[5]

$$\tan\left(\frac{\Omega R}{c}\right) = \frac{\Omega R}{c} \tag{40}$$

or

$$\tan\left(\frac{\Omega R}{c}\right) = \frac{\frac{\Omega R}{c}}{1 - \frac{\Omega^2 R^2}{c^2}} \tag{41}$$

does not change with time.

Note that every root of equation (40) is placed at the number line between two neighbouring roots of equation (41) and vice versa. One can show that the distance between these neighbouring spherical surfaces tends to $\frac{c\pi}{2\Omega}$ when $R \to \infty$. Let us also draw attention to an interesting fact that at the surfaces of the spheres of radius (40) only the magnetic field is present, and the electric field at these surfaces does not exist. It follows directly from equation (25) for $\alpha = 0$.

3.2. Analysis of the Poynting vector's field corresponding to the wave field (25)

The Poynting vector corresponding to the wave field (25) is

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{B} = \frac{\mathcal{D}^2}{8\pi}\left[\frac{\Omega\alpha^2\sin(2\theta)}{r^5}e_\theta - \frac{\Omega\alpha\gamma\sin^2\theta}{r^5}e_r\right]\sin(2\Omega t). \tag{42}$$

Let us calculate the total momentum and the angular momentum of the electromagnetic field (25) within a sphere of arbitrary radius r with the centre in the coordinate origin. Because the Poynting vector is proportional to the vector of the density of momentum at the same point, we can just calculate the integral of the Poynting vector over the volume of the sphere.

It is easy to calculate this integral if we express spherical system coordinates by Cartesian system coordinates:

$$\boldsymbol{e}_r = \mathbf{i}\sin\theta\cos\varphi + \mathbf{j}\sin\theta\sin\varphi + \mathbf{k}\cos\theta$$

and

$$\boldsymbol{e}_\theta = \mathbf{i}\cos\theta\cos\varphi + \mathbf{j}\cos\theta\sin\varphi - \mathbf{k}\sin\theta.$$

Thus, integrating (42) over the volume of the sphere we obtain

$$\int\!\!\int\!\!\int \mathbf{S}r^2\sin\theta\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\varphi = -\frac{\mathcal{D}^2 4\pi\Omega\sin(2\Omega t)}{32\pi}\mathbf{k}\int \frac{\alpha^2}{r^3}\sin^4\theta\Big|_0^\pi\,\mathrm{d}r = 0. \tag{43}$$

It means that the total momentum of the electromagnetic field (25) in a volume bounded by an arbitrary sphere with a centre at the coordinate origin *is zero* at any time. Analogously, one can show that the total angular momentum of this field configuration is zero.

[5] It follows from $\alpha = 0$ and $\gamma = 0$ respectively.

Let us now find the *loci* where the Poynting vector is zero at any instant of time. It follows from equation (42) that the conditions when the Poynting vector is zero are

$$\alpha^2 \sin(2\theta) = 0 \qquad \text{and} \qquad \alpha\gamma \sin^2\theta = 0. \tag{44}$$

From the first equation of conditions (44) we have the following possibilities:

(i) $\alpha = 0$. This automatically satisfies both conditions (44). From $\alpha = 0$ we obtain the equation

$$\tan\left(\frac{\Omega r}{c}\right) = \frac{\Omega r}{c}. \tag{45}$$

Hence, the *loci* for case (i) are spheres whose radii satisfy equation (45).

(ii) $\sin(2\theta) = 0$. This means that θ can be 0, $\frac{\pi}{2}$ or π.

(ii-1) If θ is 0 or π, in this case both equations fulfil the conditions (44). So the *locus* is the Z axis.

(ii-2) If $\theta = \frac{\pi}{2}$, this gives us two possibilities in order to satisfy the conditions (44): *either* $\alpha = 0$ (this is case (i), see above) *or* $\gamma = 0$. From the last we have

$$\tan\left(\frac{\Omega r}{c}\right) = \frac{\frac{\Omega r}{c}}{1 - \frac{\Omega^2 r^2}{c^2}}. \tag{46}$$

So the *loci* corresponding to the case $\theta = \frac{\pi}{2}$, and $\gamma = 0$ are rings in the plane XY with radii satisfying equation (46). Note that at all points of these *rings* the magnetic field is *zero*.

Now we consider spheres whose equators are the mentioned *rings*. These spheres are defined by the condition $\gamma = 0$. One can see from equation (42) that at these surfaces the Poynting vector at all points has tangential components only. Due to this fact the conservation of the energy within spheres of radii (41) becomes clearer.

Thus, the adjusted total in looking for the *loci* where the Poynting vector for the field (25) is zero at any instant of time is

> *Locus 1*: Axis Z. We call this axis the *magnetic axis* because an electric field does not exist there.
>
> *Locus 2*: Rings in the plane $z = 0$ with radii satisfying equation (46). We call these rings *electric rings* because a magnetic field does not exist there.
>
> *Locus 3*: Spheres with centres at the origin with radii satisfying equation (45). We call these spheres *magnetic spheres* because an electric field does not exist on them.

In order to elucidate the results of the last analysis better, let us adduce the graphic (figure 1) where the distribution of the Poynting vector field is shown.

We consider this distribution, for example, in the plane $x = 0$ (because of the axial symmetry of the energy density and energy-flux density distribution it is sufficient to consider this cross-section only).

We call spheres whose equator is the *electric ring* E-spheres. We call the *magnetic spheres* M-spheres. In figure 1, one can see the vertical *magnetic axis* (coinciding with the Z-axis), the first E-sphere, the first M-sphere and the second E-sphere at a given instant of time. Within E-spheres the total electromagnetic energy is conserved because the energy-flux vector at the surface of this sphere has a tangential component only. The energy transfers along this surface from pole to equator (*electric ring*) and after a certain period[6] of time reverses movement. Within the first E-sphere the energy transfers from the *magnetic axis* to the *electric ring* and after a certain time returns.

[6] This period is defined by the function $\sin(2\Omega t)$ from equation (42).

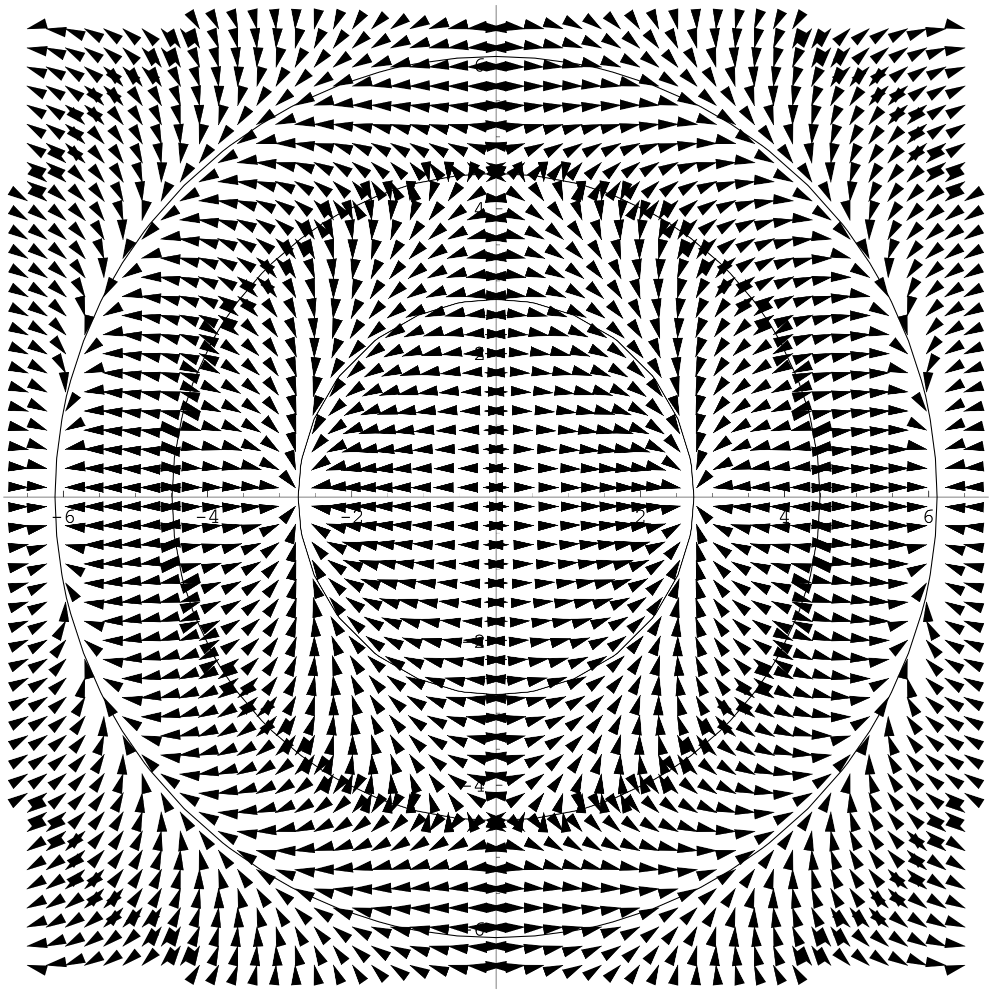

Figure 1. Poynting's vector field distribution for a given instant of time in the plane $x = 0$, the Y-axis is the abscissa and the Z-axis is the ordinate. Here $c = 1$, $\omega = 1$.

The Poynting vector is zero at every point of the first M-sphere, so the energy within this sphere is conserved too. One can see that the energy transfers from the surface of the first *magnetic sphere* to the *electric rings* of the first and the second E-spheres. An analogous exchange of the energy takes place between the next E- and M-spheres.

We once more emphasize that the Poynting vector field takes opposite directions with time, due to the existence of the function $\sin(2\Omega t)$ in equation (42).

As a further demonstration we adduce here the graphic (figure 2) of the cross-section of the Poynting vector field in the plane $z = 0$.

Finally, we adduce here the common graphic (figure 3) of the cross-section ($x = 0$) of the surfaces where the energy density is constant and the first M-sphere, first and second E-spheres.

We emphasize that these surfaces do not deform, do not displace and do not rotate with time in vacuum.

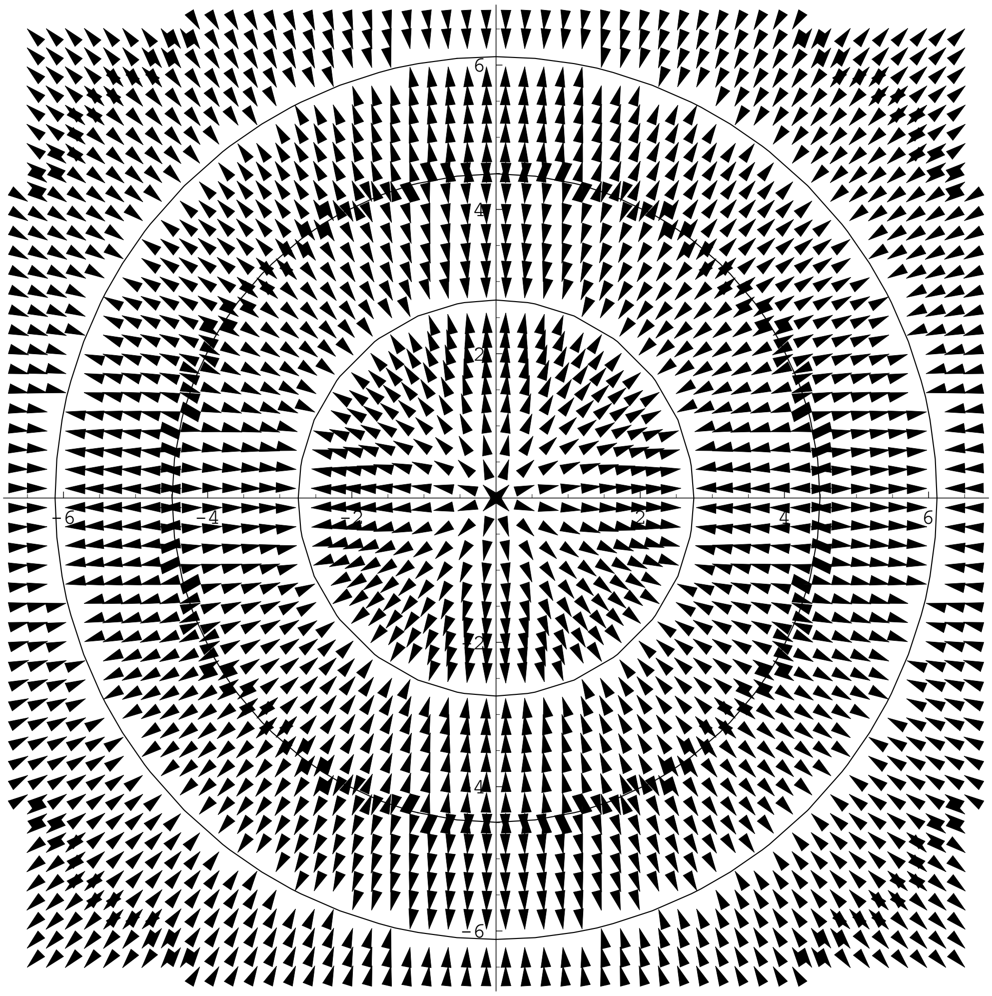

Figure 2. The Poynting vector field distribution for a given instant of time in the plane $z = 0$, the X-axis is the abscissa and the Y-axis is the ordinate.

4. Discussion

Thus, we obtained a stationary *free* electromagnetic field which can be consequence of some interference processes. Why can one speak here about interference? Actually, we see that in this electromagnetic formation, surfaces (discontinuous curves in figure 3) and points (on the Z-axis) where the energy density is *constant* exist. From this one can surmise that these surfaces and points are nodes of waves. It is also well known that standing electromagnetic waves are a result of interference processes.

Of course, the solution of the *free* Maxwell equations corresponding to these ball-like electromagnetic formations was obtained for vacuum. However, if we recall that in air, the values $\varepsilon = 1$, $\mu = 1$, we can be practically sure that the solution (25) is valid for air, taking into account that air does not have free charges and currents. So it is easy to draw an analogy between our solution and Kapitsa's hypothesis about the *interference nature* of ball lightning [4]. Actually, the electric field of electromagnetic waves which 'voyage' within M-spheres

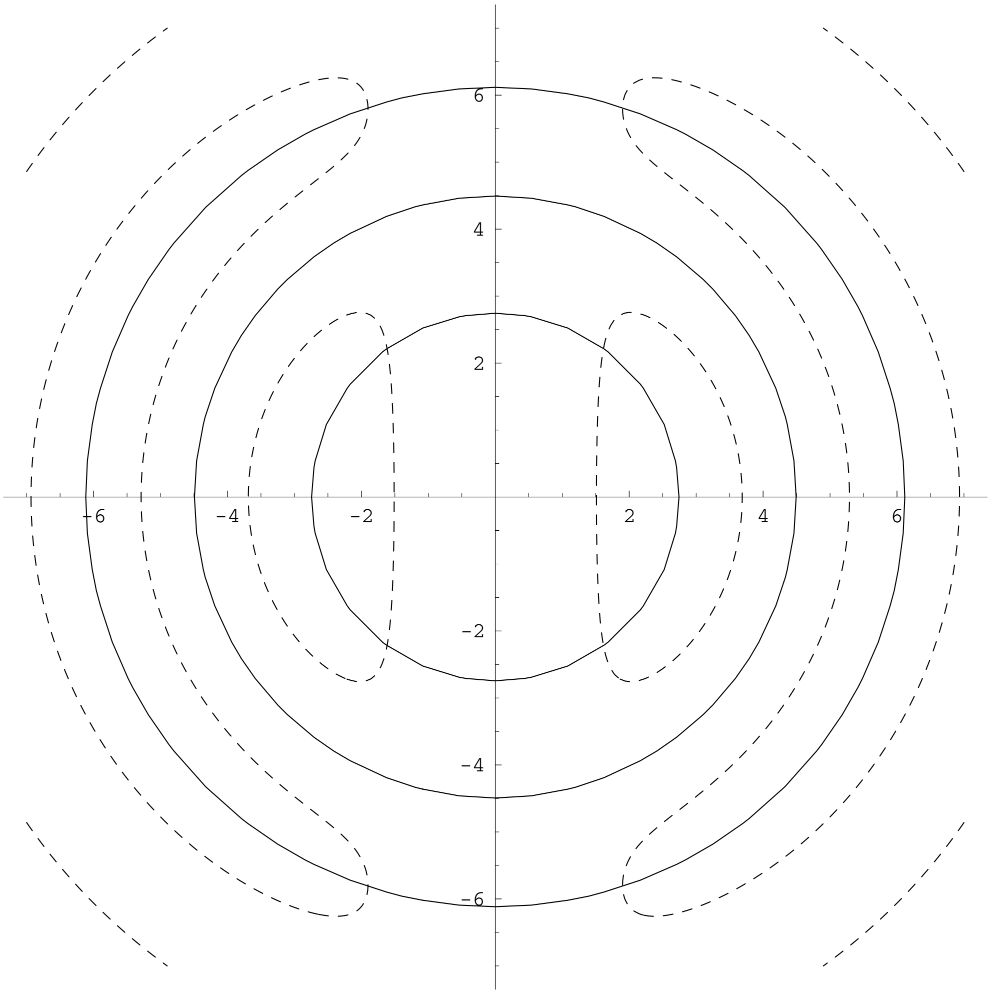

Figure 3. Cross-section of the surfaces of the constant energy density and first M-sphere and first E-spheres in the plane $x = 0$, the Y-axis is the abscissa and the Z-axis is the ordinate. Here $c = 1, \omega = 1$.

and especially the electric field of the aforementioned *electric rings* have to ionize the air converting it to plasma. The size of the critical region of ionization is defined by the radius of the *magnetic sphere*, in which the density energy is still adequate for ionizing air. This ultimate *magnetic sphere* in turn plays the role of a magnetic trap for plasma confinement. One can indeed see from equation (36) that the energy density within the magnetic spheres decreases as $\frac{1}{r^2}$. It means that at a certain distance the energy density is less than the critical value which is necessary to ionize the air. This condition has to define the radius of the ultimate magnetic sphere within which conditions of ionization still exist. Taking into account this limited value of the radius of this ultimate magnetic sphere one can speak about fireballs.

It goes without saying that it is just our hypothesis, but there is undoubtedly an analogy between Kapitsa's idea and our *ball-like* solutions. It should also be stated that other ball-like stable formations in the radiation field were obtained in the paper 'Is there yet an explanation of ball lightning?' by Arnhoff [5] and in the paper 'Ball lightning as a force-free magnetic knot' by Rañada *et al* [6] (see also [7]). It follows from these works that the electromagnetic energy contained in a spherical volume cannot escape (the energy corresponding to our solutions behaves in the same way). According to [5], outside this volume there is only an quasi-electrostatic field, rotating with constant angular velocity about the axis (at this point our and Arnhoff's solutions are different). In turn Rañada *et al* [6, 7] proposed ball-like electromagnetic formations as a solution based on the idea of the 'electromagnetic knot', an electromagnetic field in which any pair of magnetic lines or any pair of electric lines form a link—a pair of linked curves.

Thus the famous hypothesis of the Nobel prizewinner Kapitsa that fireballs (or ball lightning) are standing electromagnetic waves of unusual configuration as a result of some *interference* process from the day of its formulation (in 1955) has never (to the present day) received theoretical (mathematical) support. One can see that our work gives the first theoretical support to this hypothesis.

In a subsequent work we are going to research the process of the genesis of these unusual electromagnetic formations.

And in conclusion we just note that Barut was right when he claimed that '*electrodynamics and the classical theory of fields remain very much alive and continue to be the source of inspiration for much of the modern research work in new physical theories*' [8].

Acknowledgments

The authors would like to express their gratitude to Professor of School of Physics of Zacatecas University V Dvoeglazov and especially to Professors V Onoochin and G Kotel'nikov from Russia for their discussions and critical comments. We would also like to thank Annamaria D'Amore for revising the manuscript.

References

[1] Mie G 1908 *Ann. Phys., Lpz.* **25** 377
Born M and Wolf E 1999 *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light* 7th edn (Cambridge: Cambridge University Press)
[2] Chu C and Ohkawa T 1982 *Phys. Rev. Lett.* **48** 837
[3] Rodrigues W A and Maiorino J A 1996 *Random Oper. Stoch. Eqns.* **4** 355
[4] Kapitsa P L 1955 *DAN USSR* **101** 245 (in Russian)
[5] Arnhoff G H 1992 *Eur. Trans. Electr. Power Eng.* **2** 137
[6] Rañada A F, Soler M and Trueba J L 2000 *Phys. Rev.* E **62** 7181
[7] Rañada A F and Trueba J L 1996 *Nature* **383** 32
[8] Barut A O 1980 *Electrodynamics and Classical Theory of Fields and Particles* (New York: Dover)